

Exercise 14

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4 + 1} dz$$

Solution

According to part 1 of the fundamental theorem of calculus,

$$\frac{d}{dx} \int_a^x f(t) dt = f(x).$$

In order to make the upper limit a single variable, let $u = \sqrt{x}$.

$$h(x) = \int_1^u \frac{z^2}{z^4 + 1} dz$$

As a result, using the chain rule,

$$\begin{aligned} h'(x) &= \frac{d}{dx} \int_1^u \frac{z^2}{z^4 + 1} dz \\ &= \frac{du}{dx} \frac{d}{du} \int_1^u \frac{z^2}{z^4 + 1} dz \\ &= \frac{du}{dx} \left(\frac{u^2}{u^4 + 1} \right) \\ &= \left(\frac{1}{2} x^{-1/2} \right) \left(\frac{x}{x^2 + 1} \right) \\ &= \frac{x^{1/2}}{2(x^2 + 1)}. \end{aligned}$$